MATH 54 - HINTS TO HOMEWORK 12

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Here are a couple of hints to Homework 12! Enjoy! :)

Warning: This homework is also very long and very hard! I *strongly* urge you to read my PDEs-handout before tackling this problem set!

SECTION 10.4: FOURIER COSINE AND SINE SERIES

IMPORTANT NOTE: The book uses the following trick **A LOT**:

Namely, suppose that when you calculate your coefficients A_m or B_m, you get something like: $A_m = \frac{(-1)^{m+1}+1}{\pi m}$.

Then notice the following: If m is even, then $(-1)^{m+1} + 1 = 0$, so $A_m = 0$, and if m is odd, $(-1)^{m+1} + 1 = -2$, and $A_m = \frac{-2}{\pi m}$.

So at some point, you would like to say:

$$f(x) \text{``} = \text{''} \sum_{m=1, modd}^{\infty} A_m \cos(mx)$$

The way you do this is as follows: Since m is odd $m = 2k - 1$, for $k = 1, 2, 3 \cdots$, and so the sum becomes:

$$f(x) \text{``} = \text{''} \sum_{k=1}^{\infty} \frac{-2}{\pi(2k-1)} \cos((2k-1)x)$$

10.4.1, 10.4.3. π-periodic extension just means 'repeat the graph of f'.

The even-2π periodic extension is just the function:

$$f_e(x) = \begin{cases} f(-x) & \text{if} - \pi < x < 0 \\ f(x) & \text{if} 0 < x < \pi \end{cases}$$

The odd-2π periodic extension is just the function:

Date: Monday, December 5th.

$$f_o(x) = \begin{cases} -f(-x) & \text{if} - \pi < x < 0 \\ 0 & \text{if} x = 0 \\ f(x) & \text{if} 0 < x < \pi \end{cases}$$

And repeat all those graphs!

10.4.5, 10.4.7, 10.4.9. Use the formulas:

$$f(x) `` = " \sum_{m=0}^{\infty} A_m \cos\left(\frac{\pi m x}{T}\right)$$

where:

$$A_0 = \frac{1}{T} \int_0^T f(x) dx$$

$$A_m = \frac{2}{T} \int_0^T f(x) \cos\left(\frac{\pi m x}{T}\right) dx$$

10.4.11, 10.4.13. Use the formulas:

$$f(x) `` = " \sum_{m=0}^{\infty} B_m \sin\left(\frac{\pi m x}{T}\right)$$

where:

$$B_0 = 0$$

$$B_m = \frac{2}{T} \int_0^T f(x) \sin\left(\frac{\pi m x}{T}\right) dx$$

10.4.18. See next section!

SECTION 10.5: THE HEAT EQUATION

The best advice I can give you is: Read the PDE handout, specifically the section about the heat equation! It outlines all the important steps you'll need!

Also read the **important note** I wrote in the previous section!

10.5.7. Don't worry about this for the exam, but basically imitate Example 2! Your solution is

$$u(x,t) = v(x) + w(x,t)$$

where $v(x) = 5 + \frac{5x}{\pi}$ and $w(x,t)$ solves the corresponding homogeneous equation with $w(0,t) = 0, w(\pi,t) = 0$ but with $w(x,0) = \sin(3x) - \sin(5x) - v(x)$.

10.5.9, 10.5.13. Don't worry about this for the exam, but basically because we're dealing with an inhomogeneous solution, the general solution $u(x,t)$ is of the following form:

$$u(x,t) = v(x) + w(x,t)$$

where $v(x)$ is a **particular** solution to the differential equation, and $w(x,t)$ is the general solution to the **homogeneous** equation $(36), (37), (38)$ on page 671 (careful about the initial term, it's $w(x,0) = f(x) - v(x)$, not $w(x,0) = f(x)$)

To find v use formula (35) on page 671, and to find w, solve equations $(36), (37), (38)$.

Section 10.6: The wave equation

Read the PDE handout, specifically the section about the wave equation! It outlines all the important steps you'll need!

Section 10.7: Laplace's equation

Read the PDE handout, specifically the section about Laplace's equation! It outlines all the important steps you'll need!

The most important thing to remember is that when you solve for $Y(y)$, your solution might involve exponentials, i.e.

$$Y(y) = Ae^{wy} + Be^{-wy}$$

for some constants A, B, w (which might depend on w). Do **NOT** use this form! Instead, use the fact that:

$$\frac{e^w + e^{-w}}{2} = \cosh(w)$$
$$\frac{e^w - e^{-w}}{2} = \sinh(w)$$

and write:

$$Y(y) = A\cosh(wy) + B\sinh(wy)$$

where A and B might be constants different from above (but still call them A and B).

This will simplify your algebra by a **LOT**, trust me!

10.7.7. Again, don't worry about this for the exam, just use formulas $(22), (23), (24)$!